Early Adolescence/Mathematics

Component 1:
Content Knowledge

SAMPLE ITEMS

AND SCORING RUBRICS

NATIONAL BOARD
for Professional Teaching Standards®

Contents

Overview

This document provides information about the Early Adolescence/Mathematics (EA/Mathematics) Component 1 computer-based assessment. It includes sample assessment center selected response items and answer key, constructed response exercises, and the scoring rubric used to assess each constructed response exercise.

Component 1: Content Knowledge

Component 1: Content Knowledge is a computer-based assessment requiring candidates to demonstrate knowledge of and pedagogical practices for their teaching content area. Candidates must demonstrate knowledge of developmentally appropriate content, which is necessary for teaching across the full age range and ability level of the chosen certificate area.

EA/Mathematics Component 1 Computer-Based Assessment

In the EA/Mathematics Component 1 computer-based assessment, content knowledge is assessed through the completion of approximately 45 selected response items and three constructed response exercises.

EA/Mathematics Standards Measured by Selected Response Items

The EA/Mathematics selected response items focus on the following Standards:

Standards Content (Standard II)	Approximate Percentage of Selected Response Item Section*
Contexts for Mathematics • Historical Development of Mathematical Ideas • Mathematical Applications in Fields Related to Mathematics • Precise Communication of Mathematical Ideas	15%
Problem Solving and Number Sense • Numbers and Operations • Algebra and Functions • Geometry	45%
Modeling and Analysis • Trigonometry • Discrete Mathematics • Data Analysis and Statistics • Calculus	40%

* These percentages are an approximation only.

For the complete EA/Mathematics Standards, refer to **www.nbpts.org/certification/candidate-center/.**

EA/Mathematics Constructed Response Exercises

The EA/Mathematics constructed response exercises assess the following:

- **Exercise 1: Algebra and Functions**
  In this exercise, you will use your knowledge of algebra and functions to model problem situations, employ algebraic techniques and procedures, and explain a functional relationship depicted in a given situation. You will be asked to respond to three prompts.

- **Exercise 2: Geometry**
  In this exercise, you will use your knowledge of geometry to perform the transformations of dilation, reflection, rotation, and translation on a two-dimensional figure and explain how the two figures are congruent and/or similar. In addition, you will explain the volume formula of a three-dimensional figure and how it relates to the volume of a figure with the same type of base. You will be asked to respond to two prompts.

- **Exercise 3: Data Analysis and Statistics**
  In this exercise, you will use your knowledge of data analysis to provide various graphical representations and interpretations of a given set of data. You will be asked to respond to one prompt.

Each constructed response exercise will be assessed using a scoring rubric. Each EA/Mathematics Component 1 scoring rubric is derived from the Mathematics Standards for teachers of students ages 11–18+ and defines the levels of accomplished teaching that you must demonstrate.

> You should read the rubric while preparing to take Component 1 to understand how the rubric guides assessors in evaluating your responses to the constructed response exercises.

Inside This Document

This document includes the following three sections: "Sample Selected Response Items and Answer Key for EA/Mathematics Component 1," "Sample Constructed Response Exercises and Scoring Rubrics for EA/Mathematics Component 1," and "Calculator Information for EA/Mathematics Component 1."

Selected Response Section

This section includes the following:

- five sample selected response items
- answer key

Constructed Response Section

This section includes the following:

- three sample constructed response exercises
- associated scoring rubric for each exercise

Calculator Information

This section includes information about the online calculator provided as part of the assessment.

Other Important Information

Refer to the National Board website for the following:

- For information about scheduling and taking your test at the assessment center, please refer to the *Assessment Center Policy and Guidelines*.
- For a link to an online tutorial, please refer to the *Assessment Center Testing* page.
- For more information about how the assessment is scored, please refer to the *Scoring Guide*.

Sample Selected Response Items and Answer Key for EA/Mathematics Component 1

This section includes

- **sample selected response items** to help you become familiar with the content and format of the items on an actual computer-based assessment.

  Although this section illustrates some of the types of items that appear on the assessment, note that these sample items do not necessarily define the content or difficulty of an entire actual assessment.

  Please note that the selected response items cover the *entire* age range of the certificate. Be aware that you are expected to demonstrate knowledge of developmentally appropriate content across the full range of your certificate.

- an **answer key.**

Sample Selected Response Items

Standard II. Knowledge of Mathematics (Contexts for Mathematics)

1. Eudoxus of Cnidus (c. 395–342 BCE) used geometry to avoid irrational numbers by introducing a theory of proportion that did not involve using actual numbers. Instead, he inspected geometrical objects by comparing their relationship to each other while ignoring their precise measure. This method of mathematical inspection is most closely related to:

   A. algebraic reasoning.

   B. combinatorial design.

   C. inductive evaluation.

   D. logical construction.

Standard II. Knowledge of Mathematics (Problem Solving and Number Sense)

2. Proof by induction requires that if a property P satisfies a basic step for n = 1, and if the property is also true for P(n), then P(n + 1) is true. Therefore, the property is true for all values of n. The steps shown below prove by induction that the sum of the consecutive integers 1 through k is given by the formula $S_k = \dfrac{k(k+1)}{2}$.

$$S_k = \frac{k(k+1)}{2}$$

$$S_{k+1} = S_k + (k+1)$$

$$S_{k+1} = \frac{k(k+1)}{2} + (k+1)$$

$$S_{k+1} = (k+1)\left(\frac{k}{2} + 1\right)$$

Which equation identifies the final step needed to complete the proof?
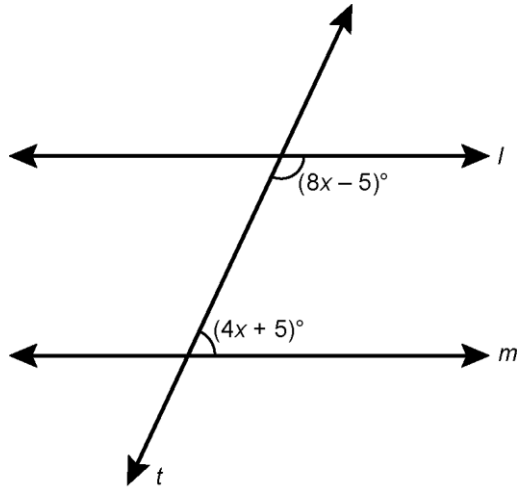
A. $S_{k+1} = \dfrac{(k+1)(k+2)}{2}$

B. $S_{k+1} = \dfrac{(k+1)(k+1)}{2}$

C. $S_{k+1} = (k+1)(k+2)$

D. $S_{k+1} = (k+1)(k+1)$

Standard II. Knowledge of Mathematics (Problem Solving and Number Sense)

3. Lines l and m are intersected by a transversal, t, as shown in the diagram below.

What must be the value of x if l is parallel to m?

A. 2.5

B. 7.5

C. 15

D. 30

Standard II. Knowledge of Mathematics (Modeling and Analysis)

4. On a sunny day, a building projects a shadow that is 56 feet long. The angle of elevation is 65° to the top of the building. Which measurement, in feet, is closest to the height of the building?

A. 24

B. 51

C. 96

D. 120

Standard II. Knowledge of Mathematics (Modeling and Analysis)

5. In which of the following sets are all the angles coterminal?

A. $\left\{ \dfrac{-13\pi}{3}, \dfrac{-7\pi}{3}, \dfrac{7\pi}{3}, \dfrac{13\pi}{3} \right\}$

B. $\left\{ \dfrac{-11\pi}{3}, \dfrac{-5\pi}{3}, \dfrac{7\pi}{3}, \dfrac{13\pi}{3} \right\}$

C. $\left\{ \dfrac{-2\pi}{3}, \dfrac{-\pi}{3}, \dfrac{\pi}{3}, \dfrac{2\pi}{3} \right\}$

D. $\left\{ \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \pi, \dfrac{4\pi}{3} \right\}$

Standard II. Knowledge of Mathematics (Modeling and Analysis)

6. Which of the following base-10 numbers represents $360_8 - 10_4$?

A. 230

B. 232

C. 236

D. 238

Answer Key to Sample Selected Response Items

Item Number	Correct Response
1	A
2	A
3	C
4	D
5	B
6	C

Sample Constructed Response Exercises and Scoring Rubrics for EA/Mathematics Component 1

This section includes

- **sample constructed response exercises** to help you become familiar with the content and format of the exercises on an actual computer-based assessment. These exercises include instructions for using the computer, stimulus materials (if applicable), and prompts requiring responses.

  Although this section illustrates some of the types of exercises that appear on the assessment, note that these sample exercises do not necessarily define the content or difficulty of the exercises on an actual assessment.

  Please note these constructed response exercises cover the **entire** age range of the certificate. Be aware that you are expected to demonstrate knowledge of developmentally appropriate content across the full range of your certificate.

- **scoring rubrics** that are used by assessors in evaluating your responses to help you understand how your responses are assessed.

Sample Exercise 1 and Scoring Rubric

Sample Exercise 1

Standard II. Knowledge of Mathematics

Exercise 1: Algebra and Functions - Candidate Name	ⓘ Time Remaining 29:31

Algebra and Functions

Introduction

In this exercise, you will use your knowledge of algebra and functions to model problem situations, employ algebraic techniques and procedures, and explain a functional relationship depicted in a given situation. You will be asked to respond to three prompts.

Criteria for Scoring

To satisfy the highest level of the scoring rubric, your responses must provide clear, consistent, and convincing evidence of the following:

- a complete and accurate graphical representation of a given set of data;
- an accurate identification of an algebraic function and a complete and accurate algebraic equation that fits the given data;
- an accurate and fully supported solution to an algebraic equation; and
- a complete and accurate modeling of a given situation and appropriate identification of its type of functional relationship.

Directions

You may preview all of the prompts by clicking the **Next** button. The **Previous** button will enable you to return to any of the prompts.

You must write your responses to all prompts for this exercise in the blue section for Exercise 1 on pages 3–12 of the Response Booklet. Graph paper is provided on pages 11–12 of this section. Your score for this exercise will be based only on what you write in the blue section for Exercise 1 of the Response Booklet.

? **Help**		◉ **Navigator**	**Next →**

Exercise 1: Algebra and Functions - Candidate Name

⏱ **Time Remaining**
29:31

1. This is Prompt 1 of 3. The second prompt appears on the next screen.

   Use the table below to respond to this prompt.

Amount of Heating Oil in Tank	
Days Since Filled (x)	**Gallons of Heating Oil** (y)
5	460
10	420
15	380
20	340
25	300
30	260
…	…

   You must address each of the following in your response to this prompt.

   - Graph the data in the table. Label axes and indicate scales.
   - Identify the type of function illustrated by the data and write an algebraic equation that expresses y as a function of x.

   Write your response in the blue section for Exercise 1 on pages 3–12 of the Response Booklet. Label your response as Prompt Number 1. Graph paper is provided on pages 11–12 of this section.

? Help   ⊙ **Navigator**   **Next →**

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⏱ **Time Remaining**
29:31

Exercise 1: Algebra and Functions - Candidate Name

2. This is Prompt 2 of 3. The third prompt appears on the next screen.

   Use the equation below to respond to this prompt.

   $$x + (x + 1) + (x + 2) + (x + 3) = 36$$

   Solve the equation for x in at least **four** steps. Justify **each** step of your solution by citing **one** of the following for each justification: addition property of equality, additive inverse, combining like terms, distributive property, factoring, multiplication property of equality, multiplicative inverse.

   Write your response in the blue section for Exercise 1 on pages 3–12 of the Response Booklet. Label your response as Prompt Number 2. Graph paper is provided on pages 11–12 of this section.

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⏱ **Time Remaining**
29:31

Exercise 1: Algebra and Functions - Candidate Name

3. This is Prompt 3 of 3.

   Use the information below to respond to this prompt.

   A teacher tells her students that she will place one penny in a jar on the first day of school and will double the amount for each day after. She tells her students she will do this for 15 days.

   - You must address each of the following in your response to this prompt.
   - Make a graph that represents the amount of pennies in the jar, y, as a function of time, x.
   - Identify the type of functional relationship depicted in the graph.

   Write your response in the blue section for Exercise 1 on pages 3–12 of the Response Booklet. Label your response as Prompt Number 3. Graph paper is provided on pages 11–12 of this section.

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Scoring Rubric for Exercise 1

The **LEVEL 4** response provides *clear, consistent, and convincing* evidence of the ability to model problem situations, employ algebraic techniques and procedures, and explain a functional relationship depicted in a given situation.

Characteristics:

- Complete and accurate graphical representation of a given set of data.

- Accurate identification of an algebraic function and complete and accurate algebraic equation that fits the given data.

- Accurate and fully supported solutions to algebraic equations.

- Complete and accurate modeling of a given situation, and appropriate identification of its functional relationship.

The **LEVEL 3** response provides *clear* evidence of the ability to model problem situations, employ algebraic techniques and procedures, and explain a functional relationship depicted in a given situation.

Characteristics:

- Accurate graphical representation of a given set of data.

- Accurate identification of an algebraic function and complete and accurate algebraic equation that fits the given data.

- Accurate solutions to algebraic equations, though lacking full support.

- A representative sketch and logical explanation of the functional relationship in a given situation.

> The **LEVEL 2** response provides *limited* evidence of the ability to model problem situations, employ algebraic techniques and procedures, and explain a functional relationship depicted in a given situation.

Characteristics:

- Incomplete and/or inaccurate graphical representation of a given set of data.

- Inaccurate identification of an algebraic function and an incomplete algebraic equation that does not fit the given data.

- Somewhat inaccurate and unsupported solutions to algebraic equations.

- An incomplete sketch and/or incomplete explanation of the functional relationship in a given situation.

> The **LEVEL 1** response provides *little or no* evidence of the ability to model problem situations, employ algebraic techniques and procedures, and explain a functional relationship depicted in a given situation.

Characteristics:

- Incomplete and inaccurate graphical representation of a given set of data.

- Inaccurate or missing identification of an algebraic function and an incomplete algebraic equation that does not fit the given data, or the equation is missing.

- Inaccurate or missing solutions to algebraic equations.

- An incomplete or missing sketch and incomplete or missing explanation of the functional relationship in a given situation.

Sample Exercise 2 and Scoring Rubric

Sample Exercise 2

Standard II. Knowledge of Mathematics

Exercise 2: Geometry - Candidate Name	ⓘ **Time Remaining** 29:31

Geometry

Introduction

In this exercise, you will use your knowledge of geometry to perform the transformations of dilation, reflection, rotation, and translation on a two-dimensional figure and explain how the two figures are congruent and/or similar. In addition, you will explain the volume formula of a three-dimensional figure and how it relates to the volume of a figure with the same type of base. You will be asked to respond to two prompts.

Criteria for Scoring

To satisfy the highest level of the scoring rubric, your responses must provide clear, consistent, and convincing evidence of the following:

- accurate transformations of dilation, reflection, rotation, and translation on a two-dimensional figure and an explanation of how the figures are congruent and/or similar; and

- a thorough and accurate analysis of the volume of a three-dimensional figure and how it relates to the volume of a figure with the same type of base.

Directions

You may preview all of the prompts by clicking the **Next** button. The **Previous** button will enable you to return to any of the prompts.

You must write your responses to all prompts for this exercise in the red section for Exercise 2 on pages 13–22 of the Response Booklet. Graph paper is provided on pages 21–22 of this section. Your score for this exercise will be based only on what you write in the red section for Exercise 2 of the Response Booklet.

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Exercise 2: Geometry - Candidate Name

⏱ **Time Remaining**
29:31

1. This is Prompt 1 of 2. The second prompt appears on the next screen.

   Refer to the two-dimensional figure described below to respond to this prompt.

   A pentagon lies in the *xy*-coordinate plane with the following coordinates as its vertices: A(2, 4), B(4, 5), C(6, 4), D(6, 2) and E(4, 1).

   You must address each of the following in your response to this prompt.

   - The pentagon is dilated by a scale factor of 1.5 with the origin as the center of the dilation. Give the coordinates of the vertices of the resulting figure.
   - The original pentagon is reflected over the line $y = -x$. Give the coordinates of the vertices of the resulting figure.
   - The original pentagon is rotated 270 degrees counterclockwise about the origin. Give the coordinates of the vertices of the resulting figure.
   - The original pentagon is translated down 2 units and left 3 units. Give the coordinates of the vertices of the resulting figure.
   - The original pentagon is reflected over one of two parallel lines. The image is then reflected over the other parallel line. Describe a single transformation that would result in a congruent image.
   - A second pentagon in the *xy*-coordinate plane has the following coordinates as its vertices: A(3, 2), B(4, 2.5), C(5, 2), D(5, 1), and E(4, 0.5). Describe a series of transformations that prove this pentagon is similar to the original pentagon.

   Write your response in the red section for Exercise 2 on pages 13–22 of the Response Booklet. Label your response as Prompt Number 1. Graph paper is provided on pages 21–22 of this section.
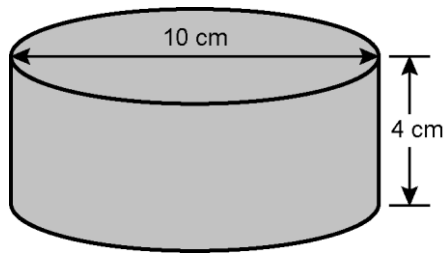
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2. This is Prompt 2 of 2.

   The three-dimensional solid shown below is a cylinder with a diameter of 10 centimeters (cm) and a height of 4 centimeters.

   There is a cone with the same base measurement and height.

   - Explain how the volume of the cylinder compares to the volume of the cone. Show all work.

   Write your response in the red section for Exercise 2 on pages 13–22 of the Response Booklet. Label your response as Prompt Number 2. Graph paper is provided on pages 21–22 of this section.

   ? Help   **⊙ Navigator**   **Next →**

Scoring Rubric for Exercise 2

The **LEVEL 4** response provides *clear, consistent, and convincing* evidence of the ability to perform the transformations of dilation, reflection, rotation, and translation on a two-dimensional object, and analyze the volume of a three-dimensional figure and how it relates to the volume of a figure with the same type of base.

Characteristics:

- Accurate transformations of dilation, reflection, rotation, and translation on a two-dimensional object and an explanation of how the figures are congruent and/or similar.

- A thorough and accurate analysis of the volume of a three-dimensional figure and how it relates to the volume of a figure with the same type of base.

The **LEVEL 3** response provides *clear* evidence of the ability to perform the transformations of dilation, reflection, rotation, and translation on a two-dimensional object, and analyze the volume of a three-dimensional figure and how it relates to the volume of a figure with the same type of base.

Characteristics:

- Mostly accurate transformations of dilation, reflection, rotation, and translation on a two-dimensional object and an explanation of how the figures are congruent and/or similar.

- An accurate analysis of the volume of a three-dimensional figure and how it relates to the volume of a figure with the same type of base.

The **LEVEL 2** response provides *limited* evidence of the ability to perform the transformations of dilation, reflection, rotation, and translation on a two-dimensional object, and analyze the volume of a three-dimensional figure and how it relates to the volume of a figure with the same type of base.

Characteristics:

- Somewhat inaccurate transformations of dilation, reflection, rotation, and translation on a two-dimensional object and an explanation of how the figures are congruent and/or similar.

- A somewhat incomplete or inaccurate analysis of the volume of a three-dimensional figure and how it relates to the volume of a figure with the same type of base.

The **LEVEL 1** response provides *little or no* evidence of the ability to perform the transformations of dilation, reflection, rotation, and translation on a two-dimensional object, and analyze the volume of a three-dimensional figure and how it relates to the volume of a figure with the same type of base.

Characteristics:

- Largely inaccurate, or missing, transformations of dilation, reflection, rotation, and translation on a two-dimensional object and an explanation of how the figures are congruent and/or similar.

- A significantly flawed, or missing, analysis of the volume of a three-dimensional figure and how it relates to the volume of a figure with the same type of base.

Sample Exercise 3 and Scoring Rubric

Sample Exercise 3

Standard II. Knowledge of Mathematics

Exercise 3: Data Analysis and Statistics - Candidate Name	⏱ Time Remaining 29:31

Data Analysis and Statistics

Introduction

In this exercise, you will use your knowledge of data analysis to provide various graphical representations and interpretations of a given set of data. You will be asked to respond to one prompt.

Criteria for Scoring

To satisfy the highest level of the scoring rubric, your response must provide clear, consistent, and convincing evidence of the following:

- a complete and accurate graphical representation of a given set of data;
- a meaningful interpretation of the data that is based on the graphical representation;
- an appropriate and accurate alternate graphical representation of the data; and
- a meaningful, accurate, and distinct interpretation of the data that is based on the alternate graphical representation.

Directions

You may view the prompt by clicking the **Next** button.

You must write your response to this exercise in the green section for Exercise 3 on pages 23–33 of the Response Booklet. Graph paper is provided on pages 32–33 of this section. Your score for this exercise will be based only on what you write in the green section for Exercise 3 of the Response Booklet.

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Exercise 3: Data Analysis and Statistics - Candidate Name

⏲ **Time Remaining**
29:31

A survey of 20 middle school students was conducted to research how much time students were spending online engaged in recreational pursuits. Students were asked, "On a typical weeknight, approximately how much time do you spend on the Internet or engaged in social media?"

Below you will find a set of data. Use the data to respond to the prompt.

Time Spent on Internet or Social Media			
2 hours	1 hour 40 minutes	80 minutes	60 minutes
90 minutes	55 minutes	1 hour	20 minutes
1 hour 15 minutes	4 hours 15 minutes	2.5 hours	None
3 hours	2 hours 10 minutes	4 hours	3.25 hours
2 hours 20 minutes	2 hours	1 hour 55 minutes	1 hour 40 minutes

You must address each of the following in your response.

- Create a boxplot of the data and provide a numerical analysis of the data that includes all of the following: minimum, first quartile, median, third quartile, maximum, and mean.

- Explain why the mean and median are or are not the same.

- Provide an analysis indicating two values 50% of the data lie between.

- Create **one** alternate graphical representation of the data. Provide an interpretation of the data revealed by this new graphical representation.

Write your response in the green section for Exercise 3 on pages 23–33 of the Response Booklet. Graph paper is provided on pages 32–33 of this section.

? Help

⊙ **Navigator**   **Next →**

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Scoring Rubric for Exercise 3

> The **LEVEL 4** response provides *clear, consistent, and convincing* evidence of the ability to provide various graphical representations and interpretations of a given set of data.

Characteristics:

- Complete and accurate graphical representation of a given set of data.

- Meaningful interpretation of the data as seen through the graphical representation.

- Appropriate and accurate alternate graphical representation of the data.

- Meaningful, accurate, and fully distinct interpretation of the data as seen through its alternate graphical representation.

> The **LEVEL 3** response provides *clear* evidence of the ability to provide various graphical representations and interpretations of a given set of data.

Characteristics:

- Mostly accurate graphical representation of a given set of data.

- Meaningful interpretation of the data as seen through the graphical representation.

- Mostly accurate and appropriate alternate graphical representation of the data.

- Meaningful and distinct interpretation of the data as seen through its alternate graphical representation.

The **LEVEL 2** response provides *limited* evidence of the ability to provide various graphical representations and interpretations of a given set of data.

Characteristics:

- Somewhat inaccurate graphical representation of a given set of data.

- Somewhat irrelevant interpretation of the data as seen through the graphical representation.

- Some inappropriate or inaccurate alternate graphical representation of the data.

- Some inaccuracies or irrelevancies or nondistinct interpretation of the data as seen through its alternate graphical representation.

The **LEVEL 1** response provides *little or no* evidence of the ability to provide various graphical representations and interpretations of a given set of data.

Characteristics:

- Inaccurate graphical representation of a given set of data.

- Irrelevant interpretation of the data as seen through the graphical representation.

- Inappropriate or inaccurate alternate graphical representation of the data.

- Inaccurate or irrelevant or nondistinct interpretation of the data as seen through its alternate graphical representation.

Calculator Information for EA/Mathematics

Component 1

An online scientific calculator is available to you for this assessment. It is similar to the Texas Instruments handheld TI-30XS scientific calculator.

To access the calculator, click on the calculator icon located in the upper left corner of the screen. A pop-up window containing the calculator will appear. You can reposition the calculator by placing your cursor in the blue area above the calculator and dragging the window to the location of your choice.

Use the numbers on the keyboard and/or point and click with the mouse to enter your computations into the on-screen calculator. When you are finished, close the calculator by clicking the button in the upper right corner of the calculator.

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